Two puzzles in probability

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National Lottery,
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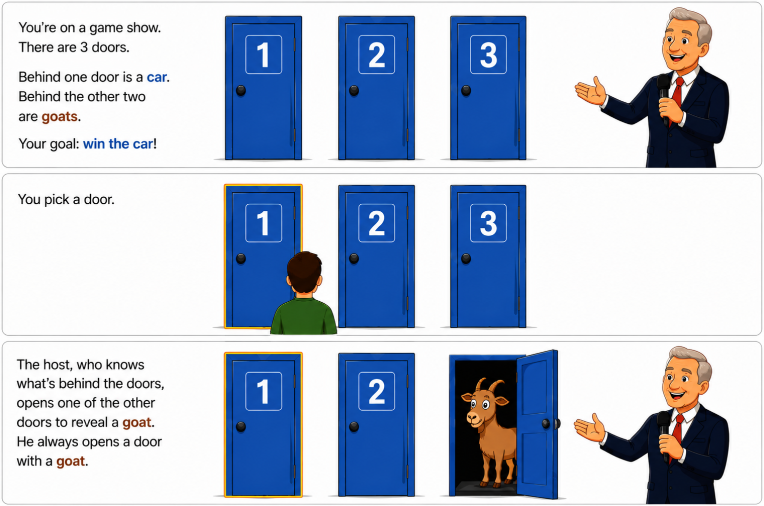
Slides available at
https://michael-mcauley.github.io

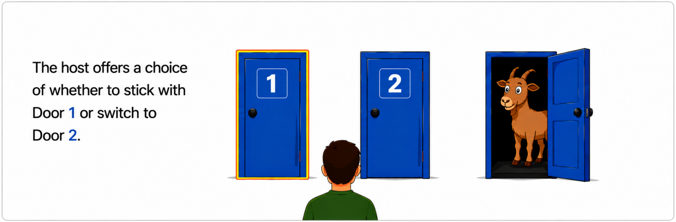
Puzzle 1

The Monty Hall Problem

The Monty Hall Problem: rules of the game



The Monty Hall Problem: stick or switch?



Should you stick or switch?

A. Stick with Door 1
B. Switch to Door 2 ⟵ **wins $\frac{2}{3}$ of the time**
C. It makes no difference

The Monty Hall problem: controversy


Ask Marilyn™

BY MARILYN VOS SAVANT

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?
—Craig F. Whitaker, Columbia, Md.

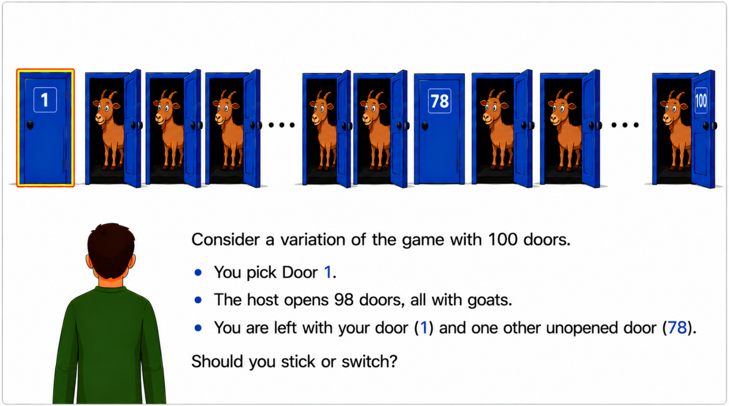
In 1990, Marilyn vos Savant gave the correct answer in her *Ask Marilyn* column: **switch**.

The magazine received $\approx 10,000$ letters disagreeing with the solution.

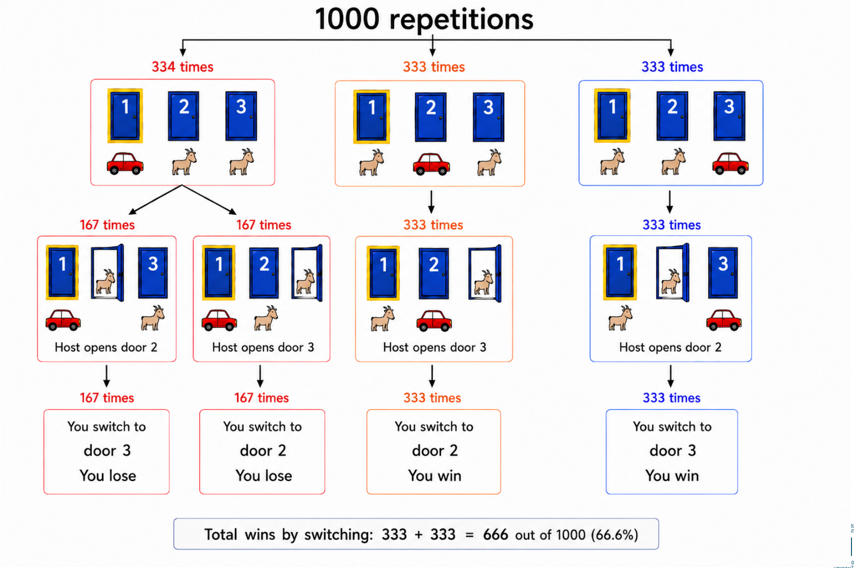
"You blew it, and you blew it big! ... There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!"

— Scott Smith, Ph.D.

Solving the Monty Hall problem: thought experiments



Solving the Monty Hall problem: natural frequencies



Solving the Monty Hall problem: Bayes' rule

Rev. Thomas Bayes
1701–1761

Pierre-Simon Laplace
1749–1827

The previous approach can be expressed more generally using a mathematical result known as **Bayes' theorem:**

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

An approachable introduction to these ideas can be found in this video by 3Blue1Brown.

The Monty Hall problem: disclaimer

"All models are wrong, but some are useful"

— George Box

- Our analysis of the Monty Hall problem makes certain assumptions, which should be checked against reality.
- In particular, our answer may change if the host can behave tactically.

Puzzle 2

The Medical Testing Paradox

Should we screen for cancer?

The case in favour:

- Lifetime risk of breast cancer for women is about 1 in 8.
- Early detection is crucial:
  - **Detection at stage 1:** 5-year survival rate $\approx$100%.
  - **Detection at stage 4:** 5-year survival rate $\approx$ 25%.
- Mammograms are affordable and widely available.

The case against:

- Screening tests are not perfectly accurate:
  - False positives
  - Overdiagnosis
  - Radiation exposure

> Routine screening is **not universally recommended** by medical organisations.

Sources: Irish Cancer Society; Cancer Research UK

Should we screen for cancer?

- To make an informed decision, we need to understand the accuracy of screening tests.
- Mammogram accuracy:
  - Patients with cancer: 87%
  - Patients without cancer 90%

Key Question: If someone tests positive, how likely is it that they actually have cancer?

1. 90% – 100%
2. 70% – 90%
3. 50% – 70%
4. Less than 50%

Answer: less than 10%.

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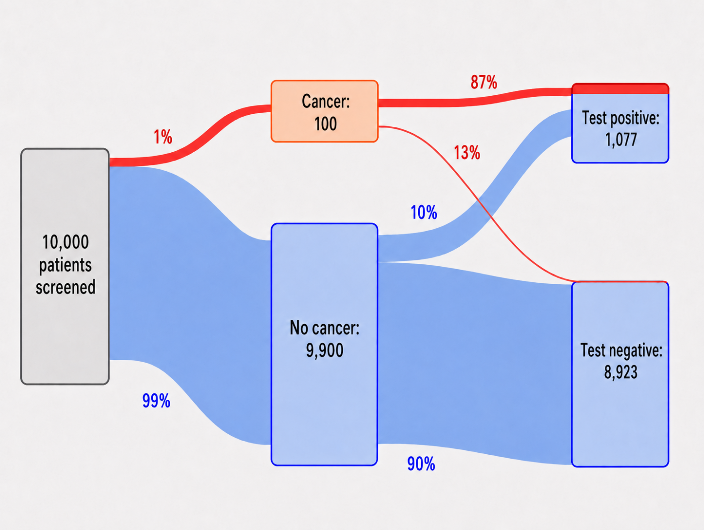
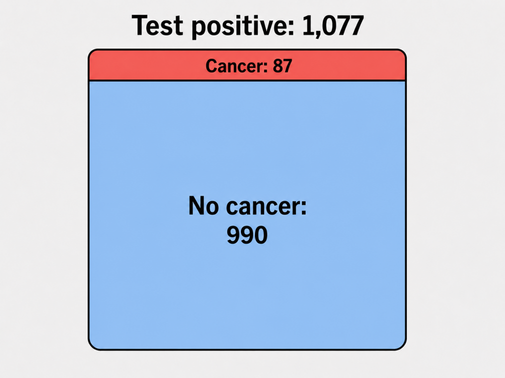
Explaining the medical testing paradox



Explaining the medical testing paradox

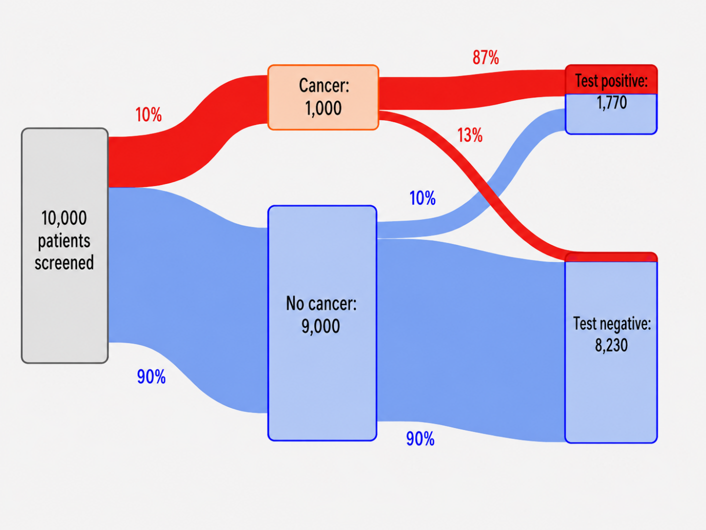


For someone who tests positive, the chance of having cancer is

$$\frac{87}{1077} \approx 8\%$$

Targeted screening

How does this change if we screen an 'at-risk' population?



Targeted screening

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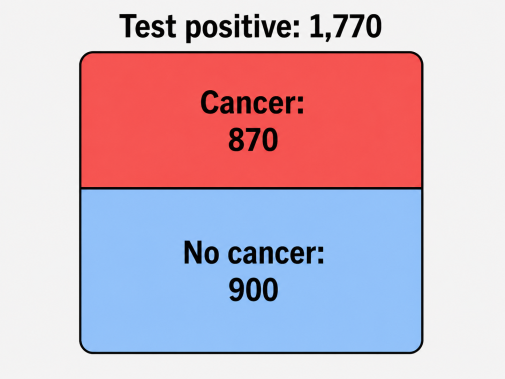


For someone who tests positive, the chance of having cancer is

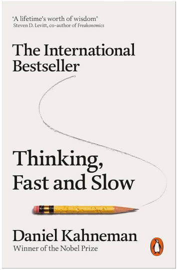
$$\frac{870}{1770} \approx 49\%$$

The medical testing paradox: disclaimer

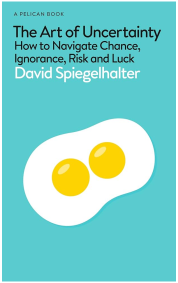
Important:

- Screening is not 'bad'!
- However, there are trade-offs which depend on the frequency of the disease.
- Real-world screening decisions are more complex.

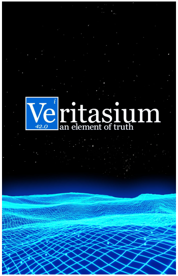
Want to learn more?



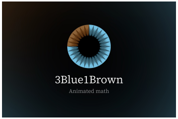
Daniel Kahneman
Thinking, Fast and Slow



David Spiegelhalter
The Art of Uncertainty



Veritasium
YouTube channel



3Blue1Brown
YouTube channel

For more on uncertainty, intuition, randomness, and mathematical thinking.

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